## CMSC 313 HW4

Due 3/7/2024 11:59pm
Please submit the completed homework through Blackboard.
There are two 4-bit unsigned values $A\left(A_{3} A_{2} A_{1} A_{0}\right)$ and $B\left(B_{3} B_{2} B_{1} B_{0}\right)$ provided as input with a maximum value of 7 . So $A_{3}$ and $B_{3}$ will be 0 .
The objective of this homework is to determine the equations for two output bits ZF and SF . The ZF bit should be 1 if $A==B$. The SF bit should be 1 if $B>A$.

The homework can be broken down into the following steps:

1. Calculate the equations for determining the 1 's complement of $B$. The inputs of this step are $\mathrm{B}_{3} \mathrm{~B}_{2} \mathrm{~B}_{1} \mathrm{~B}_{0}$ and the outputs of this step are $\mathrm{O}_{3} \mathrm{O}_{2} \mathrm{O}_{1} \mathrm{O}_{0}$. Hint: $\mathrm{O}_{0}$ only depends on $\mathrm{B}_{0}, \mathrm{O}_{1}$ only depends on $B_{1}$, etc. It is not necessary to do truth table.
2. Calculate the equations for determining -B. This is the 2's complement of $B$. The inputs of this step are $\mathrm{O}_{3} \mathrm{O}_{2} \mathrm{O}_{1} \mathrm{O}_{0}$ and the outputs of this step of $\mathrm{N}_{3} \mathrm{~N}_{2} \mathrm{~N}_{1} \mathrm{~N}_{0}$. The 2's complement of a number is found by adding 1 to the 1's complement of the number. So we are adding $\mathrm{O}_{3} \mathrm{O}_{2} \mathrm{O}_{1} \mathrm{O}_{0}$ and 0001 . Refer to the below table from the first lecture about adding 2 numbers. For the first bit, the input $A$ in the table is $\mathrm{O}_{0}$ and the input B in the table is 1 . The sum is $\mathrm{N}_{0}$. $N_{0}$ is 1 if $O_{0}$ is 0 . The carry-out $C_{1}$ is 1 if $O_{0}$ is 1 . For the second bit, the input $A$ in the table is $\mathrm{O}_{1}$ and the input B in the table is $\mathrm{C}_{1}$. The sum is $\mathrm{N}_{1} . \mathrm{N}_{1}$ is 1 if $\mathrm{O}_{1} \mathrm{C}_{1}{ }^{\prime}+\mathrm{O}_{1}{ }^{\prime} \mathrm{C}_{1} . \mathrm{C}_{2}$ is 1 if $\mathrm{O}_{1} \mathrm{C}_{1}$. Similarly, calculate the equations for $\mathrm{N}_{2}, \mathrm{C}_{3}, \mathrm{~N}_{3}$. The equations can be left as function of the inputs of this step $\left(\mathrm{O}_{3} \mathrm{O}_{2} \mathrm{O}_{1} \mathrm{O}_{0}\right)$; it is not necessary to calculate them as function of B ( $\mathrm{B}_{3} \mathrm{~B}_{2} \mathrm{~B}_{1} \mathrm{~B}_{0}$ ).

| A | B | Sum | Carry |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

3. Calculate $A-B$. The inputs for this step are $A\left(A_{3} A_{2} A_{1} A_{0}\right)$ and $-B\left(N_{3} N_{2} N_{1} N_{0}\right)$. The outputs for this step are $S\left(S_{3} S_{2} S_{1} S_{0}\right)$. For the first bit, the input $A$ is $A_{0}$ and the input $B$ is $B_{0}$. The sum $S_{0}$ is 1 if $A_{0} B_{0}{ }^{\prime}+A_{0}{ }^{\prime} B_{0}$ from the table above. The carry-out $C_{1}$ is 1 if $A_{0} B_{0}$. For the second bit, however, there are 3 inputs: $A_{1}, B_{1}, C_{1}$. We have to use the full adder to calculate the sum and carry-out. Refer to the table that we studied in the first lecture below. In our case, the input A is $A 1$, input $B$ is B 1 and carry-in input is C 1 . The sum S 1 is 1 if $\mathrm{A} 1 \oplus \mathrm{~B} 1 \oplus \mathrm{C} 1$ (i.e., A 1 xor B 1 xor C1: odd number of inputs are 1). The carry-out C2 is 1 if A1B1 + A1C1 + B1C1. Similarly calculate S2, C3, S3. The equations can be left as function of the inputs of this step: A and N.

| A | в | Carry <br> In | sum | Carry <br> Out |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

4. Calculate ZF output from $\mathrm{S}\left(\mathrm{S}_{3} \mathrm{~S}_{2} \mathrm{~S}_{1} \mathrm{~S}_{0}\right)$. Write the equation for ZF in terms of the input S . 5. Calculate SF output from $S\left(\mathrm{~S}_{3} \mathrm{~S}_{2} \mathrm{~S}_{1} \mathrm{~S}_{0}\right)$. Write the equation for SF in terms of the input S .
